A Numerical Technique for Finite Displacement Problems of Spacial Frames(空間骨組の有限変位問題に対する-数値解法)

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論　文　内　容　要　旨

1. Introduction

In spite of a large amount of studies on buckling of curved beams and finite displacement analyses of spacial frames, the lateral torsional buckling under finite displacements has not been so extensively studied yet. Finite displacement problems have difficulty in the formulation of stiffness equations, because the finite rotations are to be defined explicitly but are not linear vectors in the three-dimensional space. Under these circumstances, one of the main objectives of this study is to derive a physically clear and simply usable formulation for finite displacement problems of spacial frames including the spacial instability.

2. Governing Equation

In this chapter a governing equation of an elastic beam is derived with no restriction on the range of displacements and rotations.

3. Numerical Formulation of Spacial Frames

It is almost impossible to analyze the governing equation derived in the previous chapter as a boundary value problem to investigate large displacement behaviour and instability phenomena of spacial frames, and therefore some discretized scheme such as finite element method is generally used to convert it to the stiffness equation. Considering that the formulation of the stiffness equation by finite element method is based on the virtual work principle, we can utilize the formulation in the previous chapter. However, the governing equation is highly nonlinear equation of displacements and the substitution of displacement functions into the equation yields a complicated stiffness equation. Moreover it is difficult to define the stiffness matrix in the explicit form. In order to circumvent these difficulties, here we use a technique of the separation of rigid body displacements and employ the total-Lagrangian formulation.

In the total-Lagrangian approach, if strains are very small, by decomposition of the total finite deformation into finite rigid body rotation and small real deformation, it can be shown that the coordinate transformation and familiar elastic stiffness matrix result in a simplified stiffness equation for finite displacement problems. In the formulation, finite rotational angles are expressed by the Eulerian angles. The Eulerian angles easily constructs coordinate transformation, but the increments of them which are not defined about three axes of a rectangular Cartesian coordinate system can not be transformed into the global coordinate system to superimpose every tangent stiffness equation.

In order to make it possible to superimpose the tangent stiffness equations whose nodal rotational angles are the Eulerian angles, introducing the transformation between the infinitesimal rotational angles about three axes of the spacially fixed rectangular Cartesian coordinate system and the infinitesimal components of the Eulerian angles, we obtain the global stiffness equation in which external moments and the corresponding rotational angles can be defined as those about three axes of the spacially fixed rectangular Cartesian coordinate system.

In order to demonstrate the accuracy of the stiffness equation, the large displacement behaviour of a cantilever 45-degree bend subjected to a concentrated tip load as shown in Fig.1 is calculated. The non-dimensional tip displacement is plotted against the load parameter along with the solution by Bathe and Bolourchi. The present results agree fairly with those by Bathe and Bolourchi.

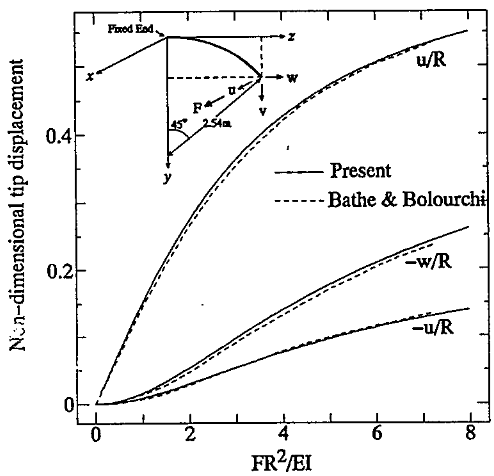

Fig. 1 Large displacement behaviour of a cantilever 45-degree bend

4. Lateral Torsional Buckling of Arches

Lateral torsional buckling of arches under uniform bending are studied by a nonlinear eigenvalue analysis and critical moments are plotted against the subtended angle in Figs.2 and 3 along with some closed form solutions. The numerical results agree well with the results by the modified Vlasov solution in which the effect of the pre-buckling in-plane displacements is taken into account.

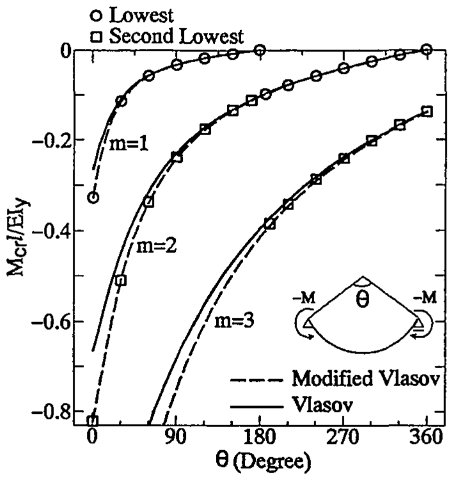

Fig. 2 Negative critical moment of circular arch

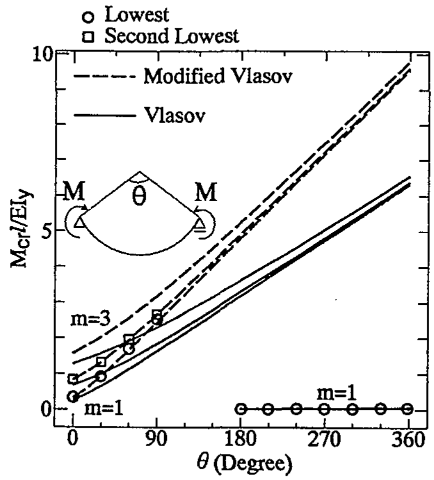

Fig. 3 Positive critical moment of circular arch

5. Experimental Study

Lateral torsional buckling experiments on acrylic arches are described. The buckling moments estimated by the Meck's method are shown in Fig.4 along with numerical and analytical solutions. The experimental results agree qualitatively well with theoretical solutions at a subtended angle smaller than 120°, but not at a subtended angle near 180°.

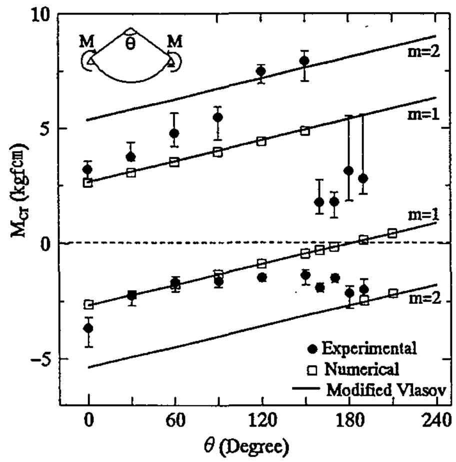

Fig. 4 Experimental results obtained by Meck's method

6. Conclusions

A simplified finite displacement formulution was proposed for spacial frames to investigate the large displacement behaviour and spacial instability phenomena within a framework of the total-Lagrangian approach based on the separation of rigid body displacements. This formulation needs no complicated step to get the stiffness equation, and it becomes more feasible than the other formulations. As one of the most typical spacial instability problems, we studied the lateral torsional buckling of arches numerically and obtained satisfactory results. Since such spacial instability problems have not been studied enough especially in the case of considering the effect of the pre-buckling in-plane displacements, the present study has valuable contributions to the field of finite displacement problems of special frames.

審査結果の要旨

空間骨組の有限変位問題及び曲がりばりの横倒れ座屈問題に関する多くの研究がなされているものの，有限変位解析に基づいた柔軟なはりの座屈解析は殆ど行われていない。また3次元有限変位解析では，有限回転角が線形ベクトルではないためにこの記述に工夫を要し，定式化は一般に煩雑になる上，導入した回転角成分と対応するモーメント成分の物理的意味が不明瞭となる場合が多い。本研究ではEuler角の微小変化成分と空間固定座標3軸回りの微小回転角成分との変換式を導入し，最終的に解くべき接線剛性方程式のモーメント外力成分と回転変位成分が共に空間固定座標3軸回りで記述できる明快な定式化を導いたもので，全編6章からなる。

第1章は序論であり，本研究の動機，必要性，目的，適用範囲と意義について述べている。

第2章では，軸力，ねじれ，2軸曲げを受ける棒部材に対する支配方程式を，変位，回転の大きさに何等の制約も設けずに定式化している。

第3章では，前章で得られた支配方程式に対する数値解析手法を，剛体変位除去の手法により定式化し，得られた数値解析手法の精度を検討するために3次元大変位問題を解析している。

第4章では，3次元面外不安定の典型的な問題としてアーチの横倒れ座屈を解析している。まず既存の解析解と比較する目的で，座屈前の面内変位の影響を無視して解析している。次に座屈前の面内変位の影響を加味して修正した解析解を用意した後，面内に大きく撓むアーチを解析しこれと比較している。

第5章では，前章の横倒れ座屈問題を実験的に検討している。前章の横倒れ座屈問題に対する解析解を検証するために，解析解の境界条件及び荷重条件を可能な限り忠実に実現した実験装置による実験結果を示し，解析解，数値解と比較検討している。

第6章は結論である。

以上要するに本論文は，有限変位解析のより物理的に明解な定式化を提案するとともに，その適用例として柔軟なアーチの横倒れ座屈を解析しかつ実験による考察を加えたものであり，構造力学・土木工学の発展に寄与するところが少なくない。

よって，本論文は博士（工学）の学位論文として合格と認める。